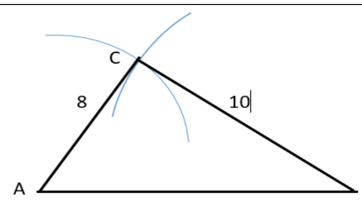
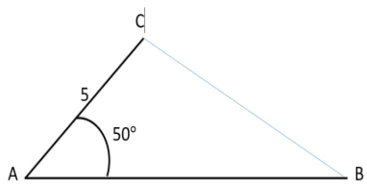
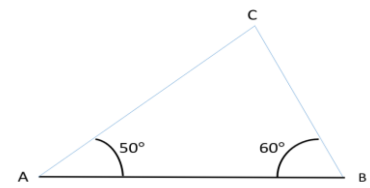
Non-right Triangle Trigonometry

Solving Non-Right Triangles: Sine and Cosine Laws

Solutions for Cases 1, 2, and 3 examples:

Case 1: (s – s - s) Ex: a = 10, b = 8, c = 12 cm	**Case 2:** (s – A - s) Ex: b = 5 cm, A = 50°, c = 12 cm	**Case 3:** (A – s - A) Ex: A = 50°, c =12 cm, B = 60°
 	 	

Ex 1: $\cos A = \frac{12^2+8^2-10^2}{2x12x8} = \frac{108}{192} = 0.56$ Therefore: $A = \cos^{-1} 0.56 = 55.77°$

The same way: $\cos B = \frac{12^2+10^2-8^2}{2x12x10} = \frac{180}{240} = 0.75$ Therefore: $B = os^{-1} 0.75 = 41.41°$

Or we can use the sine law to find the angle B, then: $C = 180 - 41.41 - 55.77$ =82.82°

Ex 2: $a^2 = 12^2 + 5^2 - 2x12x5.\cos(50°) = 91.87$ Then: $a = 9.58$ cm

Use the sine law to find the angle B: $\sin B = \frac{b\,.\ \sin A}{a} = \frac{5\,.\ \sin(50°)}{9.58} = 0.40$ Then: $B = 23.57°$

And $C = 180 - 50 - 23.57 = 106.43°$

Ex 3: We need to find the angle C first to have one angle and the opposite side to apply the Sine law:

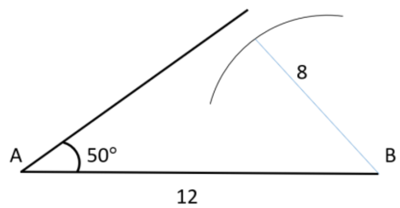
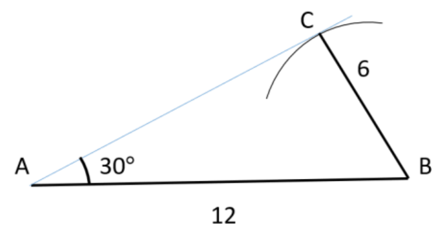
$$C = 180 - 50 - 60 = 70°$$

Now use the sine law to find sides a and b:

$\frac{sinA}{a} = \frac{sinC}{c}$ Then: $a = \frac{c\,.\sin A}{\sin C} = \frac{12\,.\sin 50}{\sin 70} = 9.78\ cm$

$\frac{sinB}{b} = \frac{sinC}{c}$ Then: $b = \frac{c\,.\sin B}{\sin C} = \frac{12\,.\sin 60}{\sin 70} = 11.06\ cm$

Solutions for Cases 4 examples

Case 4 – 1: (A - s – s) Ex: A = 50°, a = 8, c = 12 cm	**Case 4 – 2:** (A - s – s) Ex: A = 30°, a = 6, c = 12 cm
	

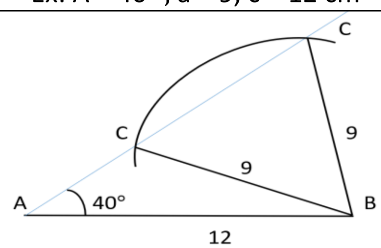
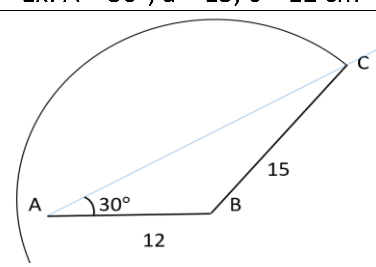
Ex 1: $a = 8$ cm, c $= 12$ cm, and A $= 50°$. Therefore:

$\sin C = \frac{12\ .\ \sin 50}{8} = 1.15$, means sin C > 1, therefore no solution.

Ex 2: $a = 6$ cm, c $= 12$ cm, and A $= 30°$. Therefore:

$\sin C = \frac{12\ .\ \sin 30}{6} = 1$, means sin C = 1, therefore C = 90°, there is one solution, a right triangle.

Therefore: $B = 90 - A = 60°$. Side b can be calculated by any of those 2 ways or by the Pythagorean formula: $b^2 = c^2 - a^2$

Case 4 – 3: (A - s – s) Ex: A = 40° , a = 9, c = 12 cm	**Case 4 – 4:** (A - s – s) Ex: A = 30°, a = 15, c = 12 cm
	

Ex 3: $a = 9$ cm, c $= 12$ cm, and A $= 40°$. Therefore: $\sin C = \frac{12\ .\ \sin 40}{9} = 0.86$, then $C_1 = 59°$

and $C_2 = 180 - 59 = 121°$. Therefore, there are 2 angles B and 2 sides b:

$B_1 = 180 - 40 - 59 = 81°$, $b_1 = 13.9\ cm$ and $B_2 = 180 - 40 - 121 = 19°, b_1 = 4.9\ cm$

Ex 4: $a = 15$ cm, $c = 12\ cm$, and $A = 30°$. Therefore: $\sin C = \frac{12\ .\ \sin 30}{15} = 0.4$, then $C_1 = 23.6°$

$C_2 = 180 - 23.6 = 156.4$, So, no second angle is possible. Then $b = 24.1\ cm$ Only one solution.

As predicted, since a>c, the second angle falls outside of side b; so, there is no second triangle.